Cambridge Assessment International Education

Cambridge IGCSE™

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4683457078

CAMBRIDGE INTERNATIONAL MATHEMATICS **0607/63**

Paper 6 Investigation and Modelling (Extended) **May/June 2020**

1 hour 40 minutes

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer both part **A** (Questions 1 to 6) and part **B** (Questions 7 to 11).
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 60.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

DC (LK/SG) 187926/2

Answer **both** parts **A** and **B**.

A INVESTIGATION (QUESTIONS 1 TO 6)

DIGITAL ROOTS (30 marks)

You are advised to spend no more than 50 minutes on this part.

This investigation is about the **digital roots** of positive integers.

To find the digital root of a positive integer, add its digits and, if necessary, the digits of the resulting number and so on until a single digit remains.

<u>Examples</u>

The digital root of 7:				= 7
The digital root of 23:	2 + 3			= 5
The digital root of 78:	7 + 8 = 15	1 + 5		= 6
The digital root of 199:	1 + 9 + 9 = 19	1 + 9 = 10	1 + 0	= 1

1 **(a)** Find the digital root of 2067.

.... [2]

(b) The digital root of 295 is 7. This can be written as D(295) = 7.

Find D(173).

.... [1]

(c) Find a 3-digit number with a digital root of 4.

.... [1]

2 **(a)** Write down the maximum value of a digital root.

.... [1]

(b) Find a number, greater than 9500, which will give this maximum value for its digital root.

.... [2]

3 **(a)** Use some values of x to find the relationship between D(x) and D($x + 9$).

.... [3]

(b) Find the relationship between D(x) and D($x + 9^n$) where n is a positive integer.

[2]

4 **(a)** Complete this table.

x	y	$D(x)$	$D(y)$	$D(x \times y)$	$D(D(x) \times D(y))$
63	101	9	2	$D(63 \times 101)$ $= D(6363)$ $= D(18)$ $= 9$	$D(9 \times 2)$ $= D(18)$ $= 9$
315	76	9	4	$D(315 \times 76)$ $= D(23940)$ = D(..................) = ...	$D(9 \times 4)$ $= D(36)$ $= 9$
253	42	1	6		$D(1 \times 6)$ $= D(6)$ $= 6$

[3]

(b) Write down an algebraic relationship between $D(x \times y)$ and $D(D(x) \times D(y))$.

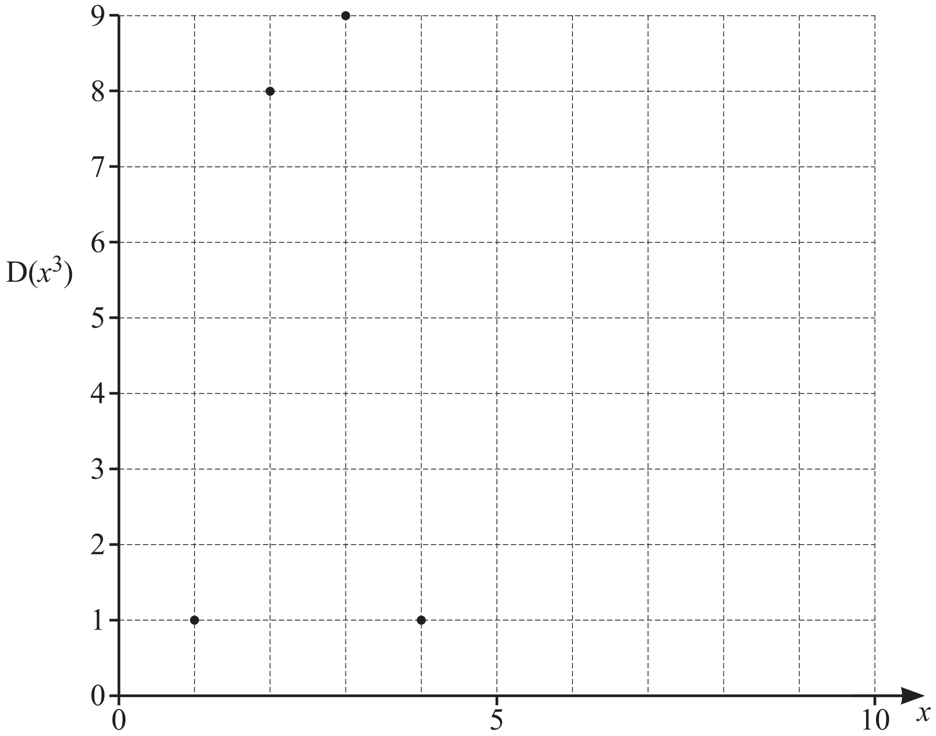
.... [1]

(c) $D(x^2) = (D(x))^2$

Is this statement correct?
Show how you decide.

.... because .... [2]

5 The diagram shows some values of $D(x^3)$ plotted against values of x from 1 to 10.



(a) Complete the diagram.

[2]

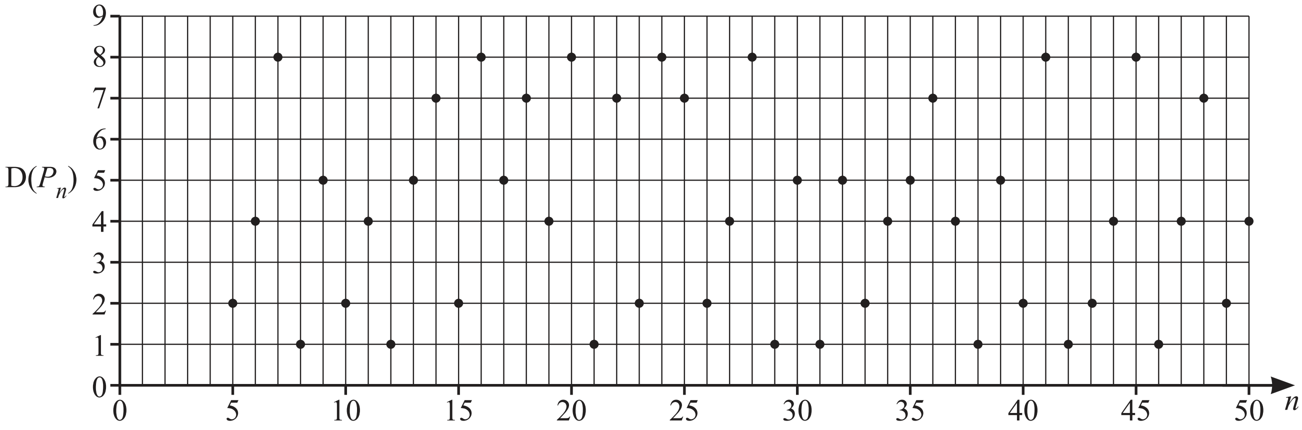
(b) Find the nth term of the sequence of values of x for which $D(x^3) = 8$.

.... [2]

(c) Use digital roots to decide whether 1 000 030 300 106 031 030 301 is a cube number.
Give a reason for your answer.

[2]

6 **(a)** P_n is the nth prime number.
The diagram shows the value of $D(P_n)$ for the 5th to the 50th prime number.



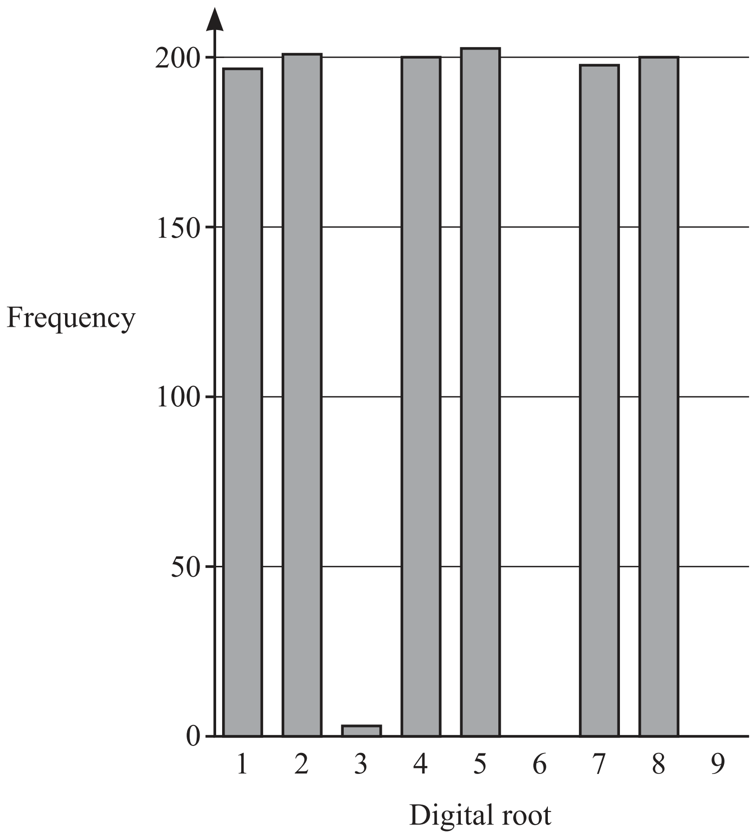
(i) Complete the diagram for the first four prime numbers.

[1]

(ii) Is it possible to use the diagram to predict $D(P_{51})$?
Give a reason for your answer.

.... [1]

(b) This diagram shows the frequency of the digital roots for the first 1200 prime numbers.



(i) Write down two observations from the diagram about the digital roots of these prime numbers.

1 ....

2 .... [2]

(ii) 4 … 27 is a 4-digit number which is not a prime number.

Use the diagram to find a possible missing digit.

.... [2]

B MODELLING (QUESTIONS 7 TO 11)

EARTHQUAKES (30 marks)

You are advised to spend no more than 50 minutes on this part.

The task is about the strength and frequency of earthquakes and the probability of their occurrence.
The strength of an earthquake is measured in magnitudes.
An increase in magnitude of 1 increases the energy released by the earthquake by a factor of 32.

Example

A magnitude 4.7 earthquake releases 32 times as much energy as a magnitude 3.7 earthquake.

7 **(a)** Write down the magnitude of an earthquake that releases 32 times the energy of a magnitude 2.5 earthquake.

.... [1]

(b) A magnitude 6 earthquake releases 30 000 units of energy.

Calculate the number of units of energy a magnitude 7 earthquake releases.

.... [2]

8 A model for the energy, E, that an earthquake releases is

$$E = g \times h^{1.5M}$$

where g and h are constants and M is the magnitude of the earthquake.

(a) An earthquake of magnitude 6 releases 30 000 units of energy.

Write an equation involving g and h.

.... [1]

(b) An earthquake of magnitude 8 releases 30 000 000 units of energy, correct to 1 significant figure.

Write an equation involving g and h.

.... [1]

(c) Use **part (a)** and **part (b)** to find

(i) the value of h,

.... [3]

(ii) the value of g.

.... [2]

(d) The magnitude of an earthquake is 6.2 .

Calculate the number of units of energy that it releases.

.... [2]

9 This table shows information about the number of earthquakes in northern Chile between April 2008 and April 2018.

Minimum magnitude (M)	Number of earthquakes (N)
3.5	2028
4.0	1912
4.5	784
5.0	230
5.5	57
6.0	14
6.5	3
7.0	0

There were a total of 2028 earthquakes with $M \geqslant 3.5$.
There were a total of $2028 - 1912 = 116$ earthquakes with a magnitude in the range $3.5 \leqslant M < 4.0$.

(a) Find the number of earthquakes in the range $5 \leqslant M < 6.5$.

.... [2]

(b) A model for this data is $N = \dfrac{k}{M}$, where k is a constant and N is the number of earthquakes with minimum magnitude M.

Is this a suitable model?
Show how you decide.

[2]

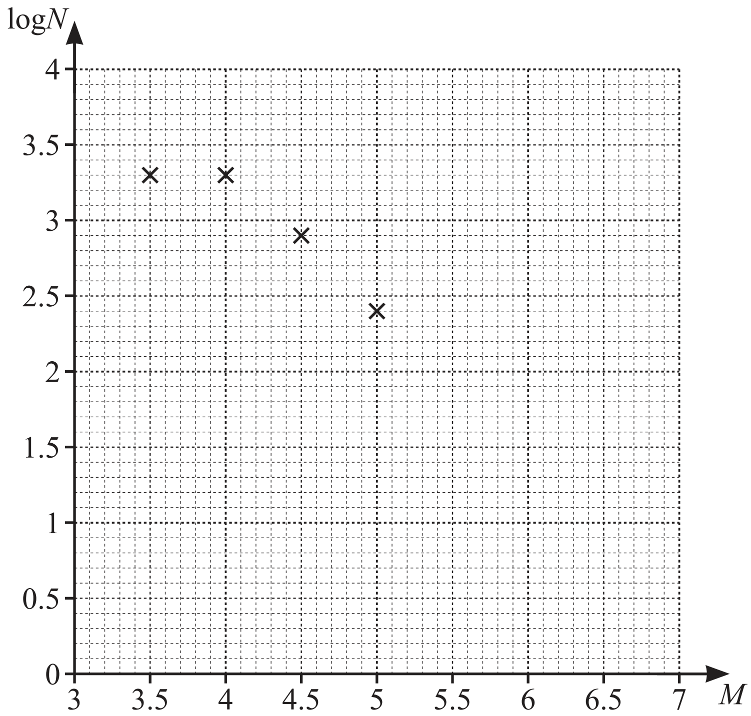
10 Another model for these earthquakes is $\log N = 7.15 + cM$ where c is a constant.

(a) Complete the table for log$N$, correct to 1 decimal place.

M	N	$\log N$
3.5	2028	3.3
4.0	1912	3.3
4.5	784	2.9
5.0	230	2.4
5.5	57	
6.0	14	
6.5	3	

[2]

(b) Complete this scatter diagram of log$N$ against M.
The first four points have been plotted for you.



[1]

(c) **(i)** The mean point is (5, 2.2).
On the diagram, draw a line of best fit. [1]

(ii) Use your line of best fit to find the value of c.

.... [2]

Question 11 is printed on the next page.

11 A model for the number of earthquakes, N, in San Francisco between 1950 and 2018 is

$$N = 10^{(6.6-0.91M)}, \text{ where } M \text{ is the minimum magnitude.}$$

(a) There were 1013 earthquakes with a minimum magnitude of 4 during this time.

Find the difference between this actual number and the number that the model predicts.

.... [2]

(b) Use the model to estimate the total number of earthquakes of any magnitude.

.... [2]

(c) **(i)** On the diagram, sketch the graph of N for $3.5 \leqslant M \leqslant 7.0$.

[3]

(ii) What effect would another earthquake of magnitude 7.0 in this period have on the graph?

.... [1]